The Loan Formula

Loans: car & student loans, credit card purchases, mortgages

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Overview

We explore the loan formula which relates P (the Principle borrowed) to the regular (re)payments PMT. Essentially you will always be finding PMT given P, or *vice versa*. The other terms in the formula will always be given.

The formula is the same each time, but the real life ramifications (and choices available to the borrower) differ for different types of loans, such as car loans, student loans, credit card purchases and mortgages.

Using this formula is calculator intensive. It's tricky to get it right due to the need to enter the various terms correctly and with appropriate parentheses.

We did car loans, student loans, and credit card purchases earlier. We review those today and then do mortgages. Mortgages present no difficulties for those who have mastered applying the formula.

The Loan Formula

$$PMT = P \times \frac{\frac{APR}{N}}{[1 - (1 + \frac{APR}{N})^{-NY}]}$$

This relates P (the Principle or amount borrowed) to the regular (re)payments PMT made N times a year.

Y is the number of years of the loan (the *term* of the loan), and the interest rate APR is always written here as a decimal.

The frequency of compounding is the same N as the frequency of repayment. If the payments are quarterly, so is the interest, and if the payments are monthly, so is the interest.

Page 250 of the textbook provides guidance as to how to use a calculator to implement this. Remember to round your final answer to the nearest penny.

Car Loans

(Example 3 on page 253) *You borrow \$6000 to buy a used car. You are given a choice of a 3-year loan at 8% or a 5-year loan at 10%. Find the monthly repayments in each case and discuss.*

Here, for the first loan conditions:

$$PMT = \$6000 \times \frac{\frac{0.08}{12}}{[1 - (1 + \frac{0.08}{12})^{-12\times 3}]}$$

This comes out to be \$188.02: the monthly repayment for $12 \times 3 = 36$ months.

This commmits you to **Total Payments** of $36 \times \$188.02 = \$6768.72$. That's \$768.72 more than you borrowed: this is interest, namely extra money you pay which the bank profits from.

Car Loans (continued)

For the second loan conditions:

$$PMT = \$6000 \times \frac{\frac{0.10}{12}}{[1 - (1 + \frac{0.10}{12})^{-12 \times 5}]}$$

This comes out to be \$127.48: the monthly repayment for $12 \times 5 = 60$ months. Smaller payments than the earlier 3-year loan, hence a better deal, right?

Wait! The 5-year loan commmits you to **Total Payments** of $60 \times \$12748 = \$7648.80$. That's \$1648.80 more than the \$6000 you borrowed.

Less pain per month, but over 5 years you are paying back more than 25% interest (as $1648.80/6000 > 1500/6000 = 25\%$.)

Car Loans (How Much Car You Afford to Borrow?)

The other use of the Loan Formula is to find P given PMT.

You can afford monthly repayments of $300. How much can you borrow to buy a car if you want to pay back over 4 years at an interest rate of 6.2% compounded monthly? Find your total repayments and the interest paid back as a percentage of the loan.

Here, the total payments are $12 \times 4 \times \$300 = \$14,400$, which will of course exceed the amount borrowed. This time, we have

$$\$300 = P \times \frac{\frac{0.062}{12}}{[1-(1+\frac{0.062}{12})^{-12\times 4}]}$$

This comes out to be

$$\$300 = P \times 0.02357683431$$

By division, we find $PM = \$12,724.35$. The interest is thus $\$14,400.00 - \$12,724.35 = \$1,675.65$. You can check that this is 13.17% of the amount borrowed.

Student Loans

Student Loans work just like Car Loans, but the amount borrowed is much higher and the loan term is typically 10 or more years.
You have a $30,000 loan, at 5.13%, to be paid off in 15 years. Find the monthly repayments, the total payments, and the interest paid.

Here,

$$PMT = \$30,000 \times \frac{\frac{0.0513}{12}}{[1 - (1 + \frac{0.0513}{12})^{-12 \times 15}]}$$

We find $PMT = \$239.27$, the repayment for $12 \times 15 = 180$ months. Hence total payments are $180 \times \$239.27 = \$43.068.60$. That's $13,068.60 more than you borrowed: dividing by $30,000 reveals you have paid back 43.56% interest!

The federal student loan interest rate for undergraduates is 4.53% for 2019-20. Check that an $80,000 loan paid back over 12 years commits you to almost $104,000 in repayments.

Credit Cards

Everytime you use your credit card to make a purchase, you are taking out a loan. The interest rates can be very high indeed, for students over 20%. As a result, the implications are serious if you don't pay back fast.

There is no fixed term (length) of the loan, and the credit card companies are happy if you stagger repayments over a long period as they make more interest off you.

They suggest minimal monthly repayments based on (1) some term they choose, and (2) the total of the PMT's for all of the purchases made the previous month, each being a loan in itself.

Since we all tend to use credit cards quite frequently, the requested monthly payments on bills are very difficult to break down. But it IS smart to make those minimum payments, as otherwise additional charges or fines can be applied. Always read the "fine" print!

Credit Cards

You spend $250 with a credit card which has an interest rate of 22%. Find the monthly repayments if you pay it off in 3 years, and also fine the total payments and the interest paid.

Here,

$$PMT = \$250 \times \frac{\frac{0.22}{12}}{[1 - (1 + \frac{0.22}{12})^{-12\times 3}]}$$

We find $PMT = \$9.55$, the repayment for $12 \times 3 = 36$ months.

Hence total payments are $36 \times \$9.55 = \$343.80$. That's \$93.80 more than you borrowed: dividing by \$250 reveals you have actually pay back an extra 37.52% (of what you borrowed) in interest. Ouch!

Do that over, this time assuming that you pay back over 6 years: the interest jumps to about 80%.

Page 255 of our text has tips on avoiding credit card trouble.

Mortgages

Mortgages are large loans repaid over very long periods. They are generally for buying houses or businesses, the term often being 20, 25 or 30 years.

Many people want a slice of the action–realty company people, lawyers, property assessors, insurance companies and more. There are down payments, closing costs and other expenses. In real life it gets very complicated, see pages 256-261 for some discussion of some of those issues (all confusingly mixed in with the basics).

In addition, as the financial landscape of the country changes, the interest rate may even go up or down; such mortgages are called adjustable rate ones.

We only consider the basic mathematics of fixed rate mortgages.

Make sure you try the relevant exercises in the text, such as #15-24, 37-40, on pages 263-264.

Mortgages

(Example 6 on page 257)

A \$100,000 mortgage comes with choices: either 8% APR for 30 years or 7.5% APR for 15 years. In each case find the monthly payments, the total payments, and the interest paid. Discuss.

For the first conditions we have

$$PMT = \$100,000 \times \frac{\frac{0.08}{12}}{[1-(1+\frac{0.08}{12})^{-12\times 30}]}$$

We find $PMT = \$733.76$, the repayment for $12 \times 30 = 360$ months.

Hence total payments are $360 \times \$733.76 = \$264,153.60$. That's \$164,153.60 more than the \$100,000 you borrowed: dividing by \$100,000 reveals you pay back 164% interest! (that's like giving them back over \$5 for every \$2 they lent you.)

Mortgages

For the second conditions we have

$$PMT = \$100,000 \times \frac{\frac{0.075}{12}}{[1 - (1 + \frac{0.075}{12})^{-12 \times 15}]}$$

We find $PMT = \$927.01$, the repayment for $12 \times 15 = 180$ months.

Hence total payments are $180 \times \$927.01 = \$166,861.80$. That's \$66,861.80 more than you borrowed, and it's 66.86% interest.

Clearly the second option is better: you save almost \$100,000.

What's the catch?

Smart mortgages options

The 30-year mortgage only commits you to $733.76 a month.

The 15-year mortgage commits you to $927.01 a month.

What would you do it you were only sure you could only afford $850 a month and you had to pick one of these mortgages?

The safest choice would be the 30-year one with the lower payments. But you will be in debt for most of your working life, and it will cost you (almost) an EXTRA $100,000 in interest.

If you go for the 15-year one, with the higher payments, you take a risk. You might pay the required $927.01 per month for a few years but if you fall short for a few months in a row, then “your” house might be reposessed by the bank.

Is there a third option? Yes!

Smart mortgages options

The REALLY SMART thing to do is to sign the paperwork for the longterm low monthly payment mortgage, which is a very serious legally binding contract, but actually pay off more every month! ("As if you were legally committed to a mortgage with higher monthly payments.")

While the interest rates just discussed were not the same, if you tried to pay about $900 a month and you succeeded most of the time (thereby "overpaying") you would in effect reduce the debt to zero in less than 20 years.

The bank has to sweat more to keep recalculating what you still owe, but that's their problem and they are legally obliged to do it and to get it right.

This is an option most people don't know about.