Comparing Normal Curve Values Using z-scores

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KEY POINT: *z-scores allow us to "compare apples and oranges", by "putting on a level playing field" numbers from that arise in different contexts.*

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KEY POINT: *z-scores allow us to "compare apples and oranges", by "putting on a level playing field" numbers from that arise in different contexts.* We just compare the appropriate z-scores.

Again assume Bio test scores are $B = N(75,5)$ and Chem test scores are $C = N(80,4)$. Your friend Danielle got 74 on Bio and 81 on Chem, which test did she do better on?

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Now consider your cousin Ed: he got 72 on Bio and 78 on Chem, which test did he do better on?

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Comparing z-scores, we see that Ed also did better on Chem: -0.5 is a bigger z-score than -0.6. (Chem score is less bad than the Bio)

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Assume the price of a certain typle of Mercedes is $M = N(35,3)$ and the price of a certain typle of Audi is $A = N(26,4)$ (both in thousands of dollars).

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The Audi is a better deal, because it has a lower z-score. You're overpaying (compared to the average) in both cases, but you're overpaying by more for the Merc, so that's a worse deal!

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If you pay \$30K for the Audi, then $z_A = \frac{30-26}{4} = 1$. That's exactly 1 std dev above the mean price (84th percentile).

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If you pay \$33K for the Merc, then $z_M = \frac{33-35}{3} = -0.6667$. That's exactly 2/3 of a std dev below mean price (unknown percentile). So the answer is 2/3 of a std dev below the mean price for the Audi, namely $\$26K - (2/3)\$4K = \$23.3333K$. Note that indeed $a = 23.3333$ does yield $z_A = \frac{23.3333-26}{4} = -0.6666$.

Paying at the same level means having equal z-scores, suggesting another way to do this problem: set the two z-scores equal (using A for the to-be-found Audi price) and solve for A using algebra.

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Write

$$\frac{A-26}{4} = \frac{33-35}{3}$$

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$$\frac{A-26}{4} = \frac{33-35}{3} = -0.6667.$$

So, $A - 26 = -(0.6667)(4)$

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Write

$$\frac{A-26}{4} = \frac{33-35}{3} = -0.6667.$$

So, $A - 26 = -(0.6667)(4)$ and $A = 26 - (0.6667)(4) = 23.3333$.

Comparing Heights–and saying goodbye to easy numbers

Assume that in Georgia adult men's heights (in inches) are N(69.2,3.1) and that adult women's heights are N(64.5,2.6).

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1. Who's taller, relative to their gender group, a 66 inch tall woman or a 71.5 inch tall man?

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1. Who's taller, relative to their gender group, a 66 inch tall woman or a 71.5 inch tall man?

The woman's z-score is $z_W = \frac{66-64.5}{2.6} = 0.5769$.

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The woman's z-score is $z_W = \frac{66-64.5}{2.6} = 0.5769$. The man's z-score is $z_M = \frac{71.5-69.2}{3.1} = 0.7419$.

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The woman's z-score is $z_W = \frac{66-64.5}{2.6} = 0.5769$. The man's z-score is $z_M = \frac{71.5-69.2}{3.1} = 0.7419$. The man is taller, relatively, his height corresponding to a higher (but unknown) percentile.

2. Repeat, for a 63.5 inch tall woman and a 67 inch tall man.

Comparing Heights–and saying goodbye to easy numbers

3. How tall would a woman need to be, to be at the same height level for women as a 6 foot tall man is for men?

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We set the z-scores equal (using W for the to-be-found Woman height) and solve for W using algebra.

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We write

$$\frac{W - 64.5}{2.6} = \frac{72 - 69.2}{3.1}$$

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We write

$$\frac{W - 64.5}{2.6} = \frac{72 - 69.2}{3.1} = 0.9032.$$

So, $W - 64.5 = (0.9032)(2.6)$

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We set the z-scores equal (using W for the to-be-found Woman height) and solve for W using algebra.

We write

$$\frac{W - 64.5}{2.6} = \frac{72 - 69.2}{3.1} = 0.9032.$$

So, $W - 64.5 = (0.9032)(2.6)$ and $W = 64.5 + (0.9032)(2.6)$ $= 66.8484$ inches.

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4. How tall would a man need to be, to be at the same height level for men as a 6 foot tall woman is for women?

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4. How tall would a man need to be, to be at the same height level for men as a 6 foot tall woman is for women?

This time we write

$$\frac{M - 69.2}{3.1} = \frac{72 - 64.5}{2.6}$$

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4. How tall would a man need to be, to be at the same height level for men as a 6 foot tall woman is for women?

This time we write

$$\frac{M - 69.2}{3.1} = \frac{72 - 64.5}{2.6} = 2.8846.$$

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4. How tall would a man need to be, to be at the same height level for men as a 6 foot tall woman is for women?

This time we write

$$\frac{M - 69.2}{3.1} = \frac{72 - 64.5}{2.6} = 2.8846.$$

Now we solve for M.

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4. How tall would a man need to be, to be at the same height level for men as a 6 foot tall woman is for women?

This time we write

$$\frac{M - 69.2}{3.1} = \frac{72 - 64.5}{2.6} = 2.8846.$$

Now we solve for M. We find that $M - 69.2 = (2.8846)(3.1)$

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4. How tall would a man need to be, to be at the same height level for men as a 6 foot tall woman is for women?

This time we write

$$\frac{M - 69.2}{3.1} = \frac{72 - 64.5}{2.6} = 2.8846.$$

Now we solve for M. We find that $M - 69.2 = (2.8846)(3.1)$ and $M = 69.2 + (2.8846)(3.1) = 78.1423$ inches.