SAT & ACT scores and baby arrivals

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Math 107-03, Spring 2020, Spelman College

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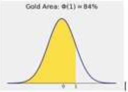
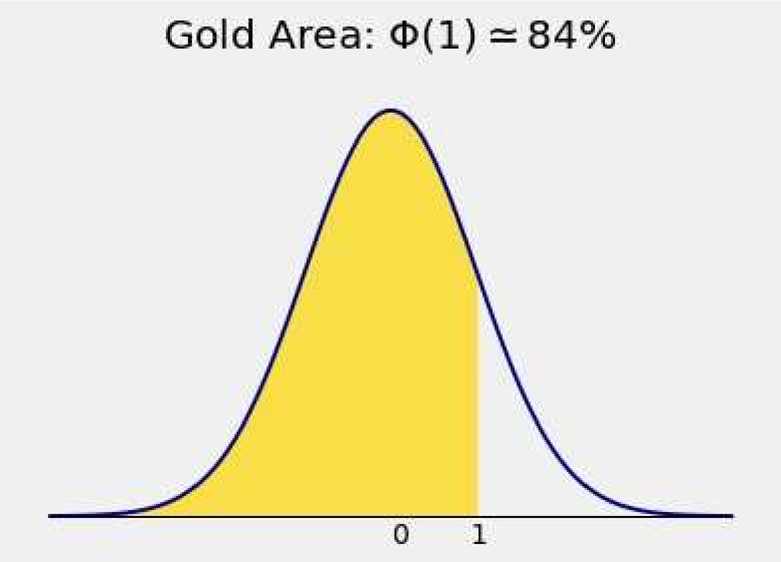
The Conversion Table

z-scores and percentiles



Z-score	Percentile	Z-score	Percentile	Z-score	Percentile	Z-score	Percentile
−3.5	0.02	−1.0	15.87	0.0	50.00	1.1	86.43
−3.0	0.13	−0.95	17.11	0.05	51.99	1.2	88.49
−2.9	0.19	−0.90	18.41	0.10	53.98	1.3	90.32
−2.8	0.26	−0.85	19.77	0.15	55.96	1.4	91.92
−2.7	0.35	−0.80	21.19	0.20	57.93	1.5	93.32
−2.6	0.47	−0.75	22.66	0.25	59.87	1.6	94.52
−2.5	0.62	−0.70	24.20	0.30	61.79	1.7	95.54
−2.4	0.82	−0.65	25.78	0.35	63.68	1.8	96.41
−2.3	1.07	−0.60	27.43	0.40	65.54	1.9	97.13
−2.2	1.39	−0.55	29.12	0.45	67.36	2.0	97.72
−2.1	1.79	−0.50	30.85	0.50	69.15	2.1	98.21
−2.0	2.28	−0.45	32.64	0.55	70.88	2.2	98.61
−1.9	2.87	−0.40	34.46	0.60	72.57	2.3	98.93
−1.8	3.59	−0.35	36.32	0.65	74.22	2.4	99.18
−1.7	4.46	−0.30	38.21	0.70	75.80	2.5	99.38
−1.6	5.48	−0.25	40.13	0.75	77.34	2.6	99.53
−1.5	6.68	−0.20	42.07	0.80	78.81	2.7	99.65
−1.4	8.08	−0.15	44.04	0.85	80.23	2.8	99.74
−1.3	9.68	−0.10	46.02	0.90	81.59	2.9	99.81
−1.2	11.51	−0.05	48.01	0.95	82.89	3.0	99.87
−1.1	13.57	−0.0	50.00	1.0	84.13	3.5	99.98

From z-scores to percentiles, visually



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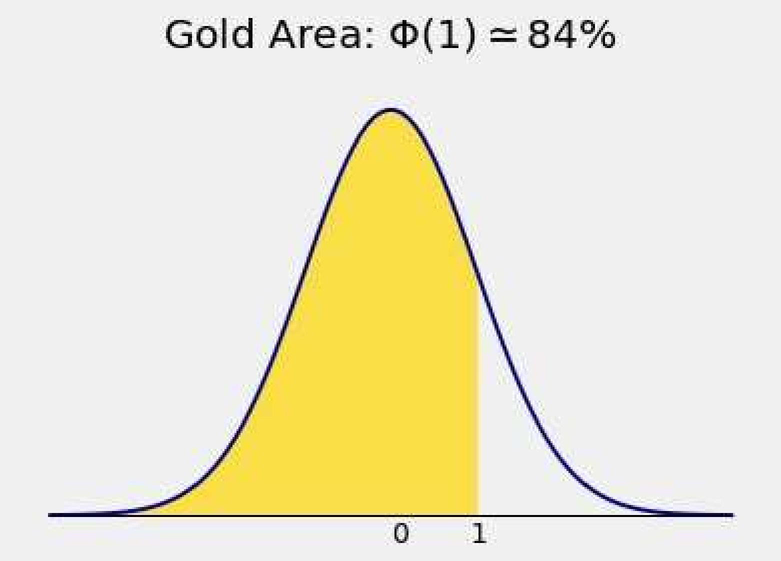
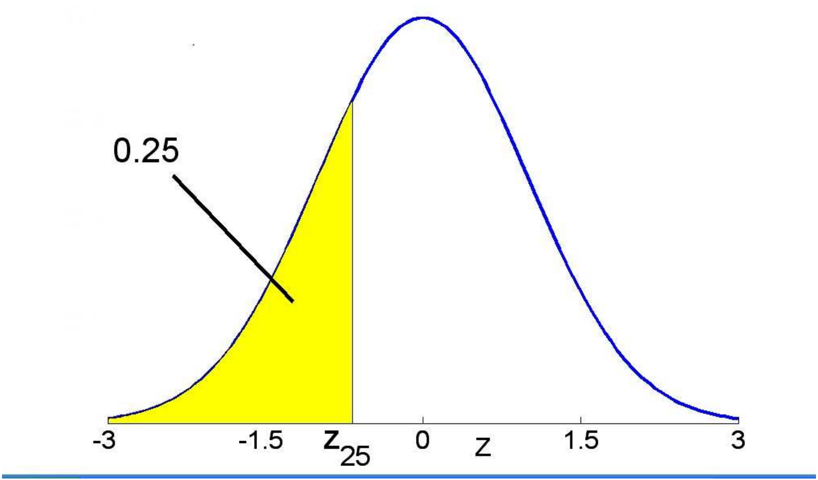


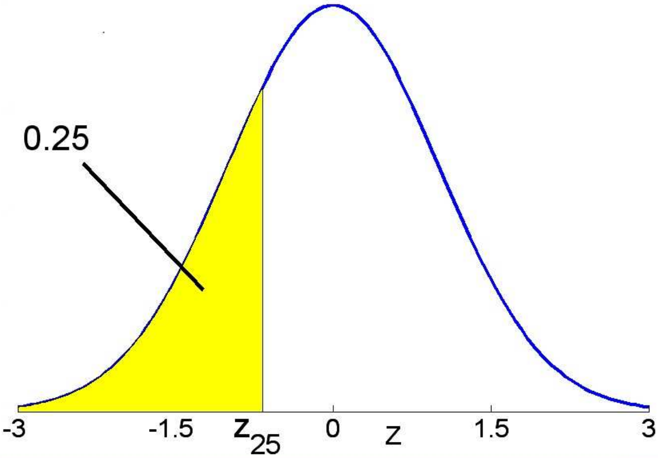
A z-score of 1 corresponds to the 84th percentile, that's the percentage of z-values equal to or less than 1.

From percentiles to z-scores, visually



What z-score corresponds to the 25th percentile?

From percentiles to z-scores, visually



What z-score corresponds to the 25th percentile? This can be read off the table, "by reading it backwards". It's near -0.65. It's even closer to -0.67 (representing $2/3$ of a std dev below the mean).

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Consider SAT scores:

About 68% of students score between 849 and 1269.

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Repeat all of the above for ACT scores.

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What ACT score is equivalent to an SAT score of 1500?

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What ACT score is equivalent to an SAT score of 1500?

What SAT score is equivalent to an ACT score of 27?

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About   % of students score between 900 and 1150.

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A student scoring 1425 is at the   th percentile.

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Scoring at the 40th percentile is getting   .

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Baby arrivals—using 68% etc rules

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About 2.5% of them arrive at least 4 weeks early early. One that arrives 4 weeks late is at the 97 and a halfth percentile.

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What percentage arrive at least 3 weeks early? Here we want $X \leq D - 21$. (Remember we must work in days, not weeks.) So,

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The table says about 69.15% arrive BEFORE they're 1 week late, so we want the other $(100 - 69.15)\% = 30.85\%$.

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This says $-\frac{5}{14} \leq Z \leq 0$, or $-0.3571 \leq Z \leq 0$.

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This says $-\frac{5}{14} \leq Z \leq 0$, or $-0.3571 \leq Z \leq 0$. Table gives about $50\% - 36\% = 14\%$.