SAT & ACT scores and baby arrivals

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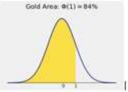
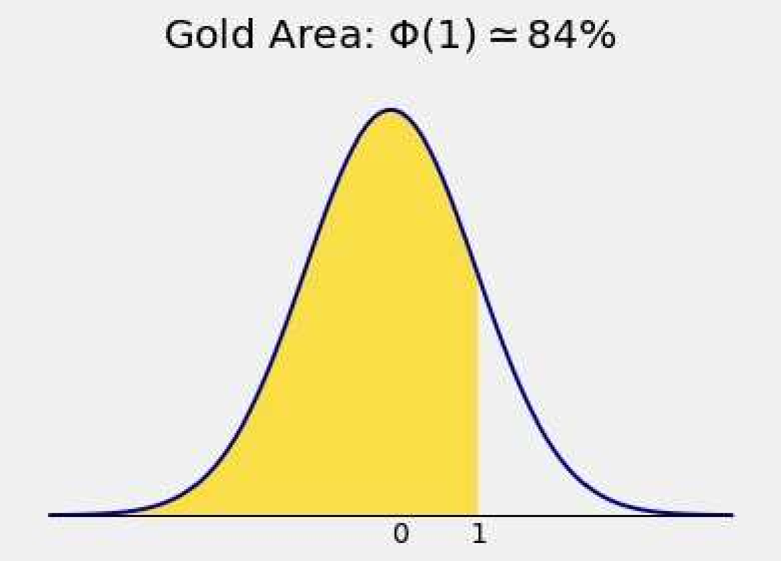
The Conversion Table

z-scores and percentiles



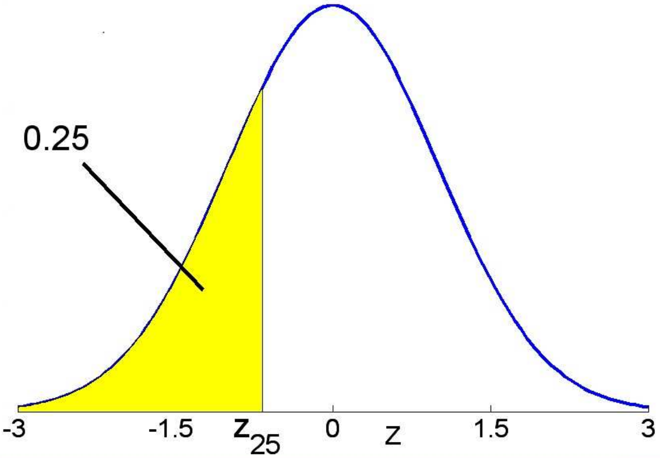
Z-score	Percentile	Z-score	Percentile	Z-score	Percentile	Z-score	Percentile
−3.5	0.02	−1.0	15.87	0.0	50.00	1.1	86.43
−3.0	0.13	−0.95	17.11	0.05	51.99	1.2	88.49
−2.9	0.19	−0.90	18.41	0.10	53.98	1.3	90.32
−2.8	0.26	−0.85	19.77	0.15	55.96	1.4	91.92
−2.7	0.35	−0.80	21.19	0.20	57.93	1.5	93.32
−2.6	0.47	−0.75	22.66	0.25	59.87	1.6	94.52
−2.5	0.62	−0.70	24.20	0.30	61.79	1.7	95.54
−2.4	0.82	−0.65	25.78	0.35	63.68	1.8	96.41
−2.3	1.07	−0.60	27.43	0.40	65.54	1.9	97.13
−2.2	1.39	−0.55	29.12	0.45	67.36	2.0	97.72
−2.1	1.79	−0.50	30.85	0.50	69.15	2.1	98.21
−2.0	2.28	−0.45	32.64	0.55	70.88	2.2	98.61
−1.9	2.87	−0.40	34.46	0.60	72.57	2.3	98.93
−1.8	3.59	−0.35	36.32	0.65	74.22	2.4	99.18
−1.7	4.46	−0.30	38.21	0.70	75.80	2.5	99.38
−1.6	5.48	−0.25	40.13	0.75	77.34	2.6	99.53
−1.5	6.68	−0.20	42.07	0.80	78.81	2.7	99.65
−1.4	8.08	−0.15	44.04	0.85	80.23	2.8	99.74
−1.3	9.68	−0.10	46.02	0.90	81.59	2.9	99.81
−1.2	11.51	−0.05	48.01	0.95	82.89	3.0	99.87
−1.1	13.57	−0.0	50.00	1.0	84.13	3.5	99.98

From z-scores to percentiles, visually



A z-score of 1 corresponds to the 84th percentile, that's the percentage of z-values equal to or less than 1.

From percentiles to z-scores, visually



What z-score corresponds to the 25th percentile? This can be read off the table, "by reading it backwards". It's near -0.65. It's even closer to -0.67 (representing $2/3$ of a std dev below the mean).

Admission Tests—SAT & ACT (using 68% etc rules)

Assume SAT scores S are N(1059,210) (on a 0–1600 scale) and assume ACT scores A are N(18.2,5.6) (on a 0–36 scale).

Consider SAT scores:

About 68% of students score between 849 and 1269. About 95% of students score between 639 and 1479. About 16% of them score above 1269. Another 16% of them score under 849. About 34% of them score between 849 and 1059. A student scoring 1269 is at the 84th percentile.

Repeat all of the above for ACT scores.

What ACT score is equivalent to an SAT score of 1500?

What SAT score is equivalent to an ACT score of 27?

Admission Tests—SAT & ACT (using table)

Assume SAT scores S are N(1059,210) (on a 0–1600 scale) and assume ACT scores A are N(18.2,5.6) (on a 0–36 scale).

Consider SAT scores:

About % of them score under 1150.

About % of them score above 1150.

About % of students score between 900 and 1150.

A student scoring 1425 is at the th percentile.

Scoring at the 40th percentile is getting .

Repeat all of the above for ACT scores.

Baby arrivals—using 68% etc rules

Assume new born babies arrive on their due date D on average, with std dev 2 weeks, so that if X represents the length of the pregnancy in days, then $X = N(D, 14)$. It's not important what D is (here it would be about 9 months expressed in days).

About 68% of babies arrive within 2 weeks of their due date. About 16% of them arrive at least 2 weeks late. About 34% of them arrive late, but no more than 2 weeks late. Another 16% of them arrive at least 2 weeks early late. A baby that arrives 2 weeks late is at the 84th percentile.

About 2.5% of them arrive at least 4 weeks early early. One that arrives 4 weeks late is at the 97 and a halfth percentile.

About 0.15% of them arrive at least 6 weeks early early.

Baby arrivals—using table

What percentage arrive at least 3 weeks early? Here we want $X \leq D - 21$. (Remember we must work in days, not weeks.) So,

$$Z = \frac{X - D}{14} \leq \frac{(D - 21) - D}{14} = -\frac{21}{14} = -1.5.$$

The table says about 6.68%.

What percentage arrive at least 1 week late? We want $X \geq D + 7$. So,

$$Z = \frac{X - D}{14} \geq \frac{(D + 7) - D}{14} = \frac{7}{14} = 0.5.$$

The table says about 69.15% arrive BEFORE they're 1 week late, so we want the other $(100 - 69.15)\% = 30.85\%$.

What percentage arrive within 3 days of their due date? We want $D - 3 \leq X \leq D + 3$. So,

$$\frac{(D-3)-D}{14} \leq \frac{X-D}{14} \leq \frac{(D+3)-D}{14}$$

This says $-\frac{3}{14} \leq Z \leq \frac{3}{14}$, namely, $-0.2143 \leq Z \leq 0.2143$.

The table gives about 42% for under -0.21 and about 58% for under 0.21 so by subtraction we get about 16%.

What percentage arrive at most 5 days early? Subtle: these are all early, just not too early! So we actually want $D - 5 \leq X \leq D$. So,

$$\frac{(D-5)-D}{14} \leq \frac{X-D}{14} \leq \frac{D-D}{14}$$

This says $-\frac{5}{14} \leq Z \leq 0$, or $-0.3571 \leq Z \leq 0$. Table gives about $50\% - 36\% = 14\%$.