Basics of Probability

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Math 107-03, Spring 2020, Spelman College

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Smart list of outcomes–ELOs

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Notice how we effectively multiplied probabilities here, this is legal because the coins are "physically independent" of each other.

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These are 36 ELOs, since for any particular pair, say $(1, 3)$, we have $P((1, 3)) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$ because the first die comes up 1 a sixth of the time, on average, and the second die comes up 3 a sixth of the time, on average.

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The totals when rolling 2 fair dice

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	1	2	3	4	5	6
1	2	3	4	5	6	7
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Note that $P(\text{don't get 6}) = 1 - P(\text{get 6})$.

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$$P(\text{not } E) = \frac{\#(\text{not } E)}{\#(S)} = \frac{\#(S) - \#(E)}{\#(S)}$$

$$= \frac{\#(S)}{\#(S)} - \frac{\#(E)}{\#(S)} = 1 - \frac{(\#E)}{(\#S)} = 1 - P(E)$$

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Let T be the total for 2 fair dice.

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