“At Least One” and Birthday Problems

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Math 107-03, Spring 2020, Spelman College

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"At Least One" Problems for Coins

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Repeat for 10 fair coins.

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It's $1 - P(\text{get no Girls})$

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Now consider 10 babies. What is $P(\text{get at least one Girl})$?

It's $1 - P(\text{get no Girls}) = 1 - \text{P(BBBBBBBBBB)} = 1 - (\frac{1}{2})^{10} = 1 - \frac{1}{1024} = \frac{1023}{1024}$.

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This technique works in other situations not based on ELOs.

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2. Now suppose you run into 50 people over the course of a week in April 2020. Assume that 4% of them are carriers of a virus. We'll assume that the other 96% are healthy (denoted by H).

What is $P(\text{you meet at least one person with the virus})$?

It's $1 - P(\text{you meet none with virus}) = 1 - \text{P(HHHH} \ldots \text{H)} =$
$1 - (0.96)^{50} = 1 - 0.1300 = 0.8700.$ That's an 87% chance of some exposure. Now try similar problems with different numbers.

Birthday Problems

Here's a famous Dilbert cartoon:

Birthday Problems

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Why is it funny?

Birthday Problems

First, we focus on days of the week. Pick a person at random, there are 7 days of the week they could have been born on:

$$\{Mon, Tue, Wed, Thu, Fri, Sat, Sun\}$$

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What is the probability that one randomly selected person was born on a Sat or Sun?

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What is the probability that one randomly selected person was born on a Sat or Sun? It's $P(\text{Sat or Sun}) = \frac{2}{7}$.

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What is the probability that one randomly selected person was born on a Sat or Sun? It's $P(\text{Sat or Sun}) = \frac{2}{7}$.

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What is the probability that one randomly selected person was born on a Sat or Sun? It's $P(\text{Sat or Sun}) = \frac{2}{7}$.

What is the probability that one randomly selected person was not born on a Mon? It's $1 - P(\text{Mon}) = 1 - \frac{1}{7} = \frac{6}{7}$.

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What is the probability that one randomly selected person *born on a weekday* was born on a Mon or a Fri?

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What is the probability that one randomly selected person *born on a weekday* was born on a Mon or a Fri? It's $\frac{2}{5}$

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What is the probability that one randomly selected person *born on a weekday* was born on a Mon or a Fri? It's $\frac{2}{5} = 0.40$.

Birthday Problems

Now pick 2 people at random.

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Now pick 2 people at random. This gives rise $7^2 = 49$ ELOs: $\{$(Mon,Mon), (Mon,Tue), ..., (Mon,Sun), ..., (Sun,Sun)$\}$.

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What's prob that both people were born on a Wed?

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What's prob that both people were born on a Wed? It's $(\frac{1}{7})^2 = \frac{1}{49}$.

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What is the probability that neither of them were born on a Tue? It's $P(\text{both born on a non Tuesday}) = (\frac{6}{7})^2 = \frac{36}{49}$.

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What is the probability that neither of them were born on a Tue? It's $P(\text{both born on a non Tuesday}) = (\frac{6}{7})^2 = \frac{36}{49}$. So the prob that at least one of them was born on a Tue is $1 - \frac{36}{49} = 0.2653$.

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The prob that they were born on different days of the week?

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The prob that they were born on different days of the week? It's $\frac{7}{7} \times \frac{6}{7} = \frac{6}{7} = 0.8571$.

Birthday Problems

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Hence, the prob that they were born on the same day of the week is $1 - P(\text{they were born on diff days}) = 1 - \frac{6}{7} = \frac{1}{7} = 0.1429$.

Birthday Problems

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What's prob that all of them were born on a Wed? It's $(\frac{1}{7})^5$.

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What's prob that none of them were born on a Tue? It's $P(\text{all born on a non Tuesday}) = (\frac{6}{7})^5 = 0.4647$. So prob at least one of them was born on a Tue is $1 - 0.4647 = 0.5373$.

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Moral: coincidences shouldn't surprise us!

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Moral: coincidences shouldn't surprise us! Some of them are very likely to occur.

Real Birthday Problems

Now pick 10 people at random.

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Hence, prob that at least two of them were born on the same day of the year is $1 - P(\text{all diff days}) = 1 - 0.8831 = 0.1169$.

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It's not to surspiring that with 10 people we don't expect some birthday coincidence. So let's increase the number of people.

The Birthday Paradox

Consider 23 randomly selected people.

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For 30 people

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