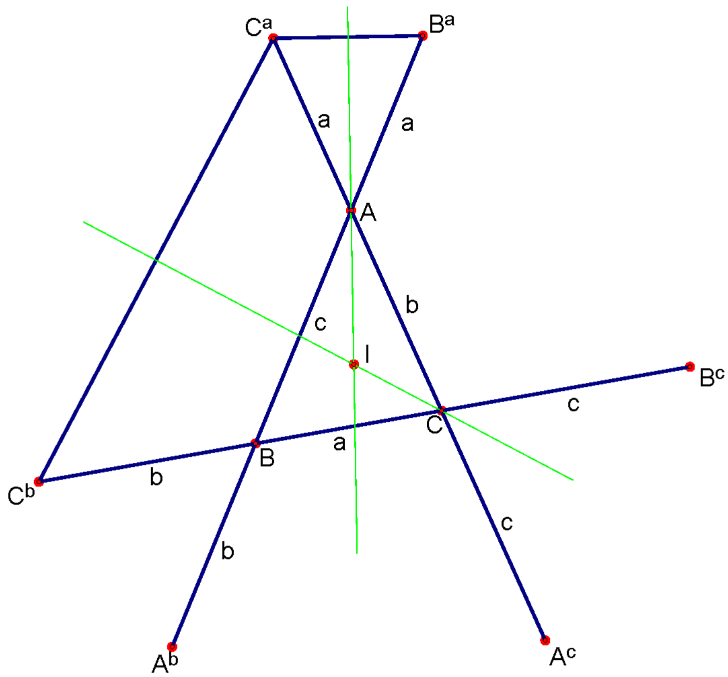
Theorem (Conway's Circle)**.** *Given a triangle ABC, with side lengths a, b, c as usual, consider a point A^c that lies on the ray AC at distance c beyond the vertex C. Points B^a, C^a, C^b, A^b and B^c have analogous definitions. Then the six points B^a, C^a, C^b, A^b, A^c and B^c lie on a circle whose center is the incenter I of triangle ABC.*



Proof. Triangles B^aAC^a and C^aCC^b are isosceles with apexes at A and C. Therefore the perpendicular bisectors of their bases are the bisectors of the angles at their apexes, namely of the angles A and C of triangle ABC. We deduce that the perpendicular bisectors of $B^a\ C^a$ and $C^a\ C^b$ meet at I. In particular, I is equidistant from the three points B^a, C^a and C^b.

A similar argument shows that I is also equidistant from the three points of $\{C^b,\ A^b,\ A^c\}$ and from the points of $\{A^c,\ B^c,\ B^a\}$. Since the three sets of points overlap, we see that I is equidistant from B^a, C^a, C^b, A^b, A^c and B^c. $\square$

EXtra-Conway Circles

John always advised looking for extra versions of theorems, especially theorems about incenters. He was surely aware that his circle has extra versions centered at the excenters E_a, E_b, E_c of triangle ABC. The EXtra-Conway circle centered at E_a passes through six points A^c, A^b, B_c, B_a, C_a and C_b, where a typical *new* point C_b is found by rotating C^b by 180° about the point B.

The following diagram shows the Conway circle and the three EXtra-Conway circles. An argument very similar to the one given above proves the assertion about the EXtra-Conway circles.