‘Make it, mathematics, stick’

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Background/Motivation

Motivation is based on three ‘resources’:

- Years of experience in lecturing, teaching, tutoring, marking assignments/exams; organising support for students and initiating support groups.
- The book “Make it stick”, (subtitled ‘The Science of Successful Learning’; authors Peter Brown, Henry Roediger and Mark A McDaniel). This has had a tremendous influence and bears out and supports ideas derived from first item.
- ‘Seven Myths About Education’, by Daisy Christodoulou.

Mathematical Education?

I am not a mathematical educationalist as such, having just one (academic) article that could be considered to be 'Mathematical Education'.

I have visited schools on numerous occasions and have talked to students and teachers therein on aspects of maths' education.

I have written for IMTA.

A number of invited articles for the IT and Indo on related topics have appeared over the years.

I have appeared on panels and been interviewed on radio on numerous occasions.

All of these have resulted in a huge correspondence from people on their worries about mathematics at all the various levels.

Make mathematics stick, please

However since reading ‘Make it stick’ I now also have about 300 pages of justification, including about 30 pages of listings of references, resources, experiments which back up my (sometimes controversial) views!

Gone mad

Educationalists, very few of whom are mathematicians, have gone mad in trying to 'reform' maths and maths teaching with the excuse that present (or past now) methods do not work. Things like: "There is a problem with x-level maths; solution is (without any doubt!!): 'change the syllabus' and all will be right."

Of course there is need for change with changing times and changing ideas and circumstances; change does not mean 'reform'.

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Primary schools: An example

A retired primary school inspector contacted me recently wanting to know if I knew anything about the new maths syllabus being drawn up for use in primary schools. I didn't! She was extremely worried.

She mentioned that the main motivating factor/research being used is that proposed by Hersh. Did I know anything about this? I didn't! But I knew who to ask!

I asked Rob Craigen who stated: "I have read Hersh's book "What is mathematics, REALLY", and I am appalled. I think it's a crime against rational thought."

He sends me a long email explaining why; this includes quotes from Martin Gardiner who had grave reservations on the work of Hersh and many arguments with him.

Rob finishes:

"Yes, educationists seem to love Hersh. That's a scary thought."

Courses on lecturing techniques for maths

In summary they don't exist! Communicating maths is so very different to communicating other areas.

HoDs in Maths Depts in UK were asked to give their opinions on such courses. The general consensus was: They are unsuitable for Maths. The report on the survey appeared in Notices of the LMS.

Required study

Unequivocally: The book ‘Make it stick’ should be required reading and required study for all such courses.

The course then is:

1. A detailed study of ‘Make it stick’.
2. Each subject group would then relate ‘Make it stick’ and possibly other learning techniques to the teaching and learning of their subject.

A detailed study of ‘Make it stick’ should be a required part of any teaching training programme.

Make mathematics stick

The book is not particularly related to mathematics.

It is so easily translated to work for learning and teaching mathematics – 'making mathematics stick'!

It has theory that is sensible, logical, well-motivated and has been shown to work!

It can be applied to mathematics with proper interpretation. To ignore its messages and techniques is to ignore the realities of learning and of learning mathematics.

Reform...

'Reform' of mathematics is often based on false premises and proposed by those who have little knowledge of mathematics. ('A little learning ...').

Change does not necessarily mean or imply 'reform'. And:

It's not all about changing the syllabus! And/or giving bonus points!

In the Irish context, these have made the situation worse, giving the illusion that something useful and successful is being done, when the opposite is the case.

Correspondents from outside Ireland maintain it's a world-wide phenomenon.

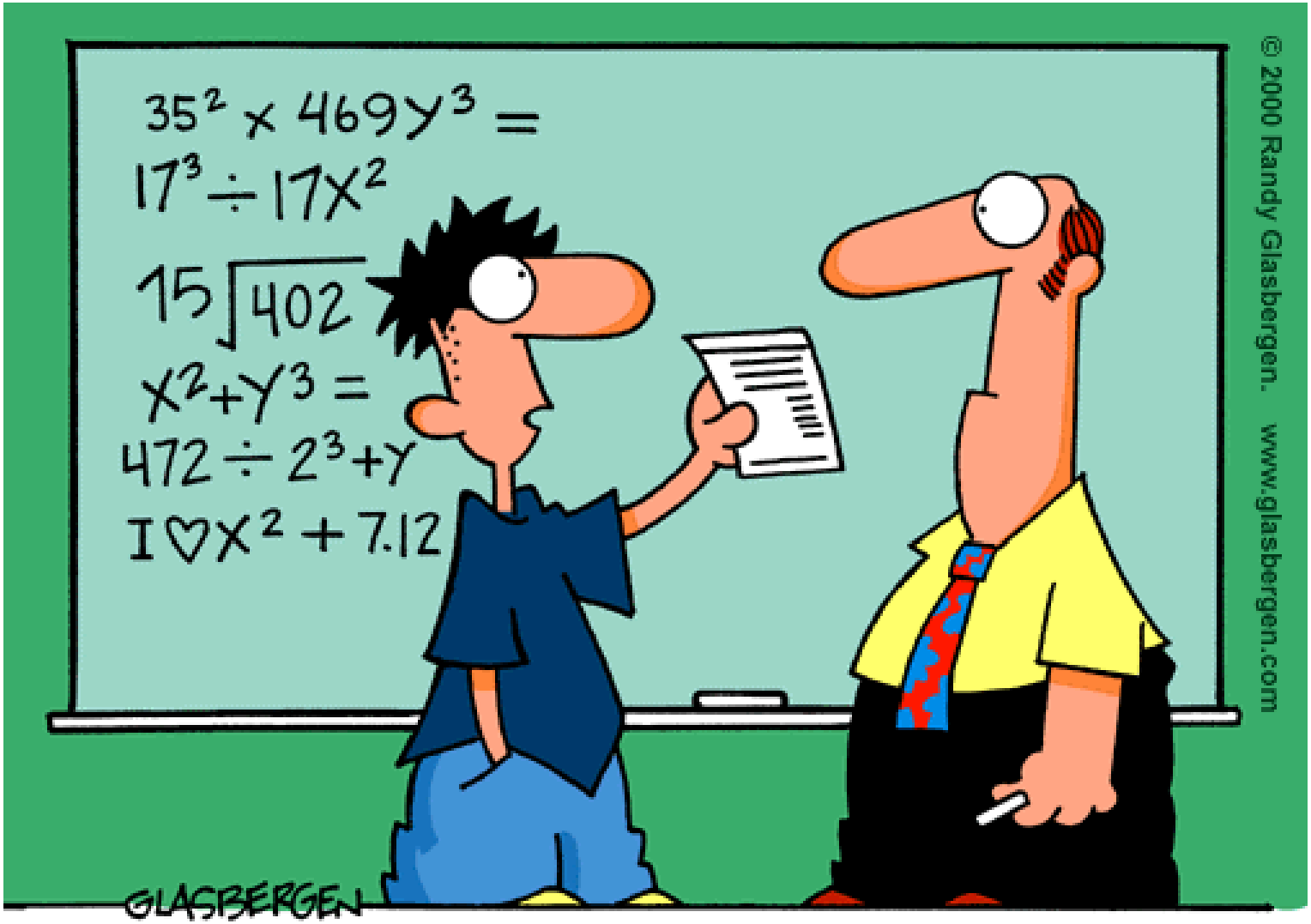
Memory: memories are made of this, you can't beat the memories ..

'We need memory in order to make progress. People misunderstand learning.'

'Not all advice is grounded in research, far from it..' So very true for maths!

'... much of what we've been doing as teachers and students isn't serving us well, but some comparatively simple changes could make a big difference..... **when learning is harder, it's stronger and lasts longer...**'

Math gene ..



"I HAD MY DOCTOR DO A D.N.A. BLOOD ANALYSIS. AS I SUSPECTED, I'M MISSING THE MATH GENE."

Born numerate, not literate

Recent research makes the striking point that we are not born to read and our brains are hard-wired to understand mathematical concepts.

We all have the maths gene and studies have demonstrated how everyone, unless he or she has some physical damage to a certain part of the brain, can understand and do maths to a reasonable level. Maths is not hard, it's just that we don't do it enough.

If people don't know a language it is easy to fool them in that language. By 'language' here is meant the language and logic of maths. The media makes numerous numerate blunders ...

Rote

"rote" means "by repetition", sharing a root with "rotation" and "rota". Memory is the seat of learning; repetition is the single most important practice for the establishment of memory.

That rote, within educationist circles, has come to mean "without understanding", is revealing in itself. Understanding requires a substrata of mastered factual knowledge.

Supplies

Memorising facts is like stocking a construction site with the supplies to put up a house.

One cannot apply what one knows in a practical manner if one does not know anything to apply.

Mastery requires both the possession of ready knowledge and the conceptual understanding of how to use it.

Testing

If we stop thinking of testing as a dipstick to measure learning, *if we think of it as practicing retrieval from learning..*, we open up another possibility: the use of testing as a tool for learning.

One of the most striking research findings is the power of active retrieval (testing) to strengthen memory, and that the more effortful the retrieval, the stronger the benefit.

In effect retrieval-testing interrupts forgetting.

Despite what research and personal experience tells us about the power of testing as a learning tool, teachers and students rarely use it as such.

It is now known from empirical research that practicing retrieval makes learning stick far better than does reexposure to the original material; the 'retrieval-practice effect'.

Strategies

There are simple and practical strategies that anyone can use:

- Various forms of retrieval practice, such as low stakes assignments combined with class tests.
- *Spacing out* practice, *interleaving* the practice of different but related topics or skill.
- Trying to solve a problem before being taught the solution.
- Distilling the underlying principles or rules that differentiate types of problems.
- Learning is an iterative process that requires that you revisit what you have learned and continually update it and connect it with new knowledge.
- Revisiting with new knowledge and new experiences and visualising and mentally rehearsing what you might do differently the next time.

Progress

In mathematics automated knowledge is critical for progress through higher levels of learning. Because of the limitations in working memory, failure to have memorised one's maths' facts leads to a log jam in the very part of one's brain that is necessary for higher processing of those cognitive skills we all agree are the goals of mathematical education.

Whitehead made this point masterfully when he said:

"It is a profoundly erroneous truism ... that we should cultivate the habit of thinking of what we are doing. The precise opposite is the case. Civilization advances by extending the number of important operations which we can perform without thinking about them."

Memory again

That memory is vital for learning is fundamental. Long-term memory is a central and dominant feature of cognition, and all aspects of seeing, hearing, thinking and understanding is ultimately dependent on, and influenced by, our memory. Critical thinking processes such as reasoning and problem-solving are intimately dependent on factual knowledge stored in long-term memory.

And yet many modern education systems, prominent educationalists and even government agencies fail to recognise this and give advice contrary to the basic facts of memory retention. Some advocates of '21st-century skills' speak disparagingly of knowledge and want to marginalise its place in the curriculum. There are some facts you simply have to learn before you can progress any further.

Facts are facts!

'Facts enable understanding and are not opposed to understanding. Our long-term memories can store a great deal of information, but our working memories are limited. What we've committed to memory helps us to solve problems.'

'Recent studies suggest that people often use the wrong strategies for learning and don't appreciate ones that work.

For example, studies show that **retrieval practice and testing** is a much more powerful learning strategy than repeated review and re-reading, and also practice is much more effective when it's broken into periods that are **spaced** out.'

"It's a fact: *when learning is harder it lasts longer*. If we don't teach powerful knowledge in schools, we end up with social inequality, because richer pupils will gain that knowledge from their parents and private tutors, whilst poorer children will not."

Mathematical learning and facts

"The project mathematics now being implemented at second level means that some of the basic knowledge requirements necessary for international-standard third-level programmes have been omitted.

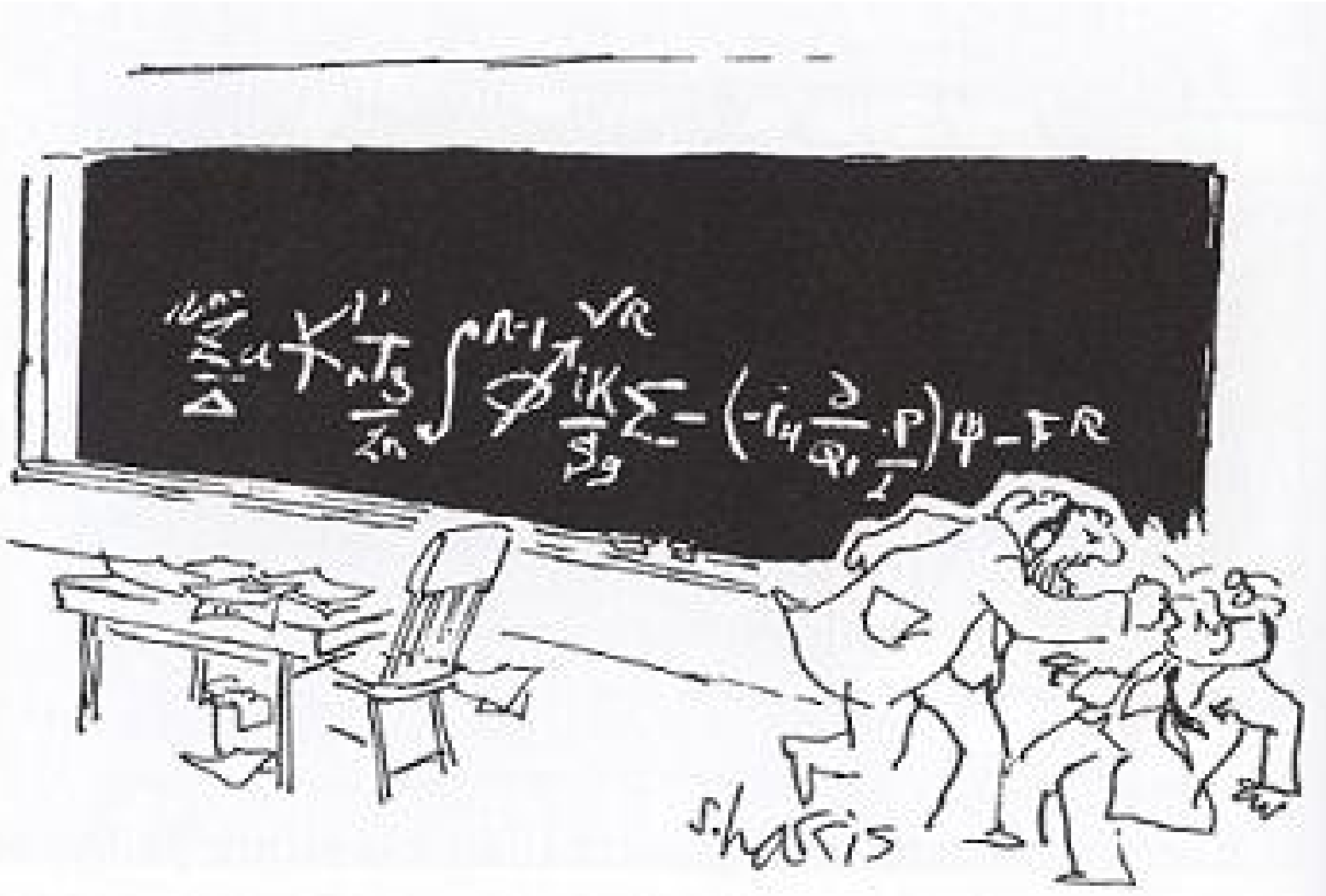
Our reputation is at stake, and our progress towards a knowledge economy has taken a backward step.

The state of maths here is vitally important in more ways than the educational."

Remember: To err is ..

but (s)he who never makes a mistake, never makes a discovery!

Proof..

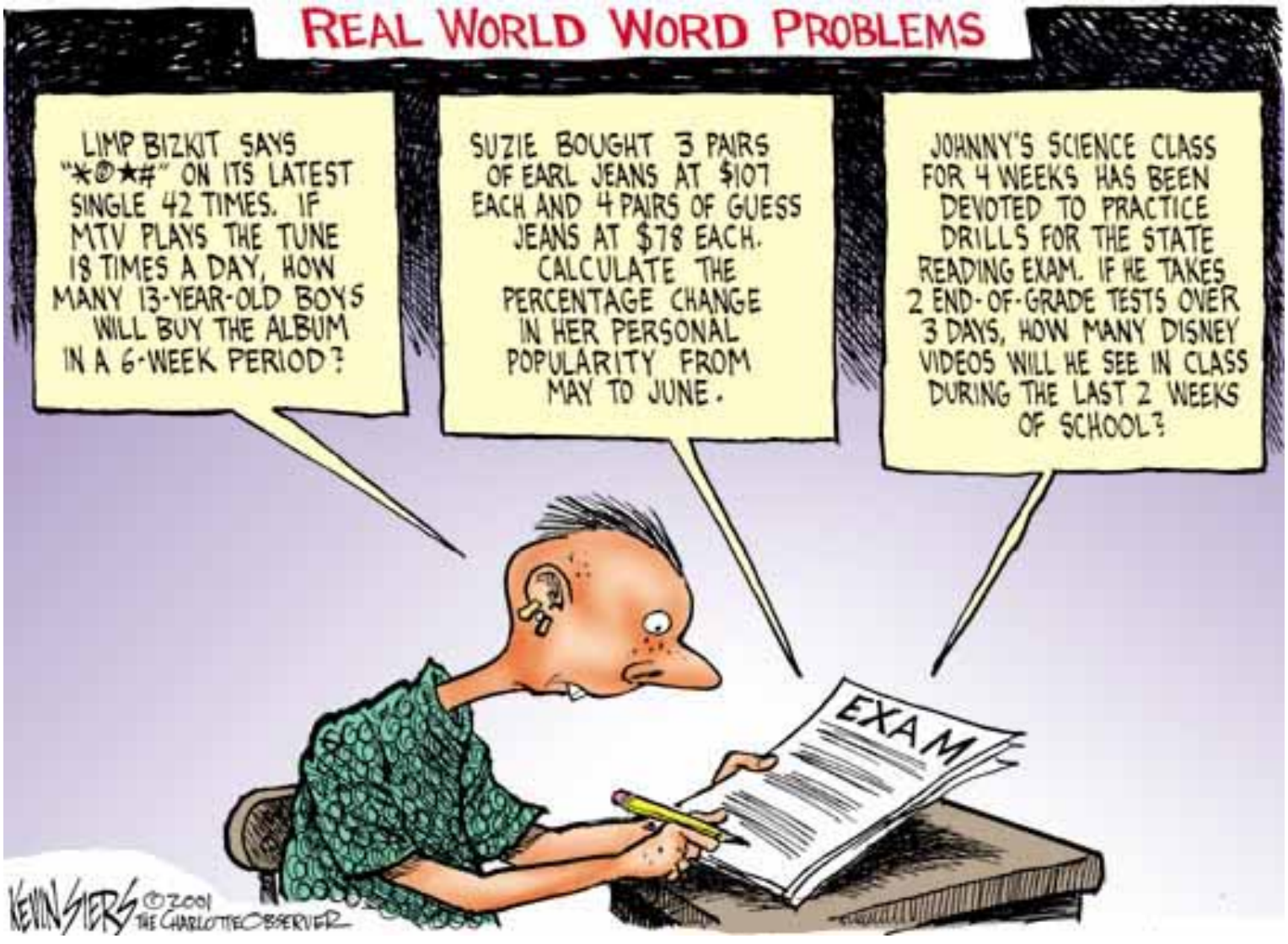


"YOU WANT PROOF? I'LL GIVE YOU PROOF!"

Learn your algebra – or else!

"Algebra class will be important to you later in life because there's going to be a test six weeks from now."

Learn your algebra – or else!



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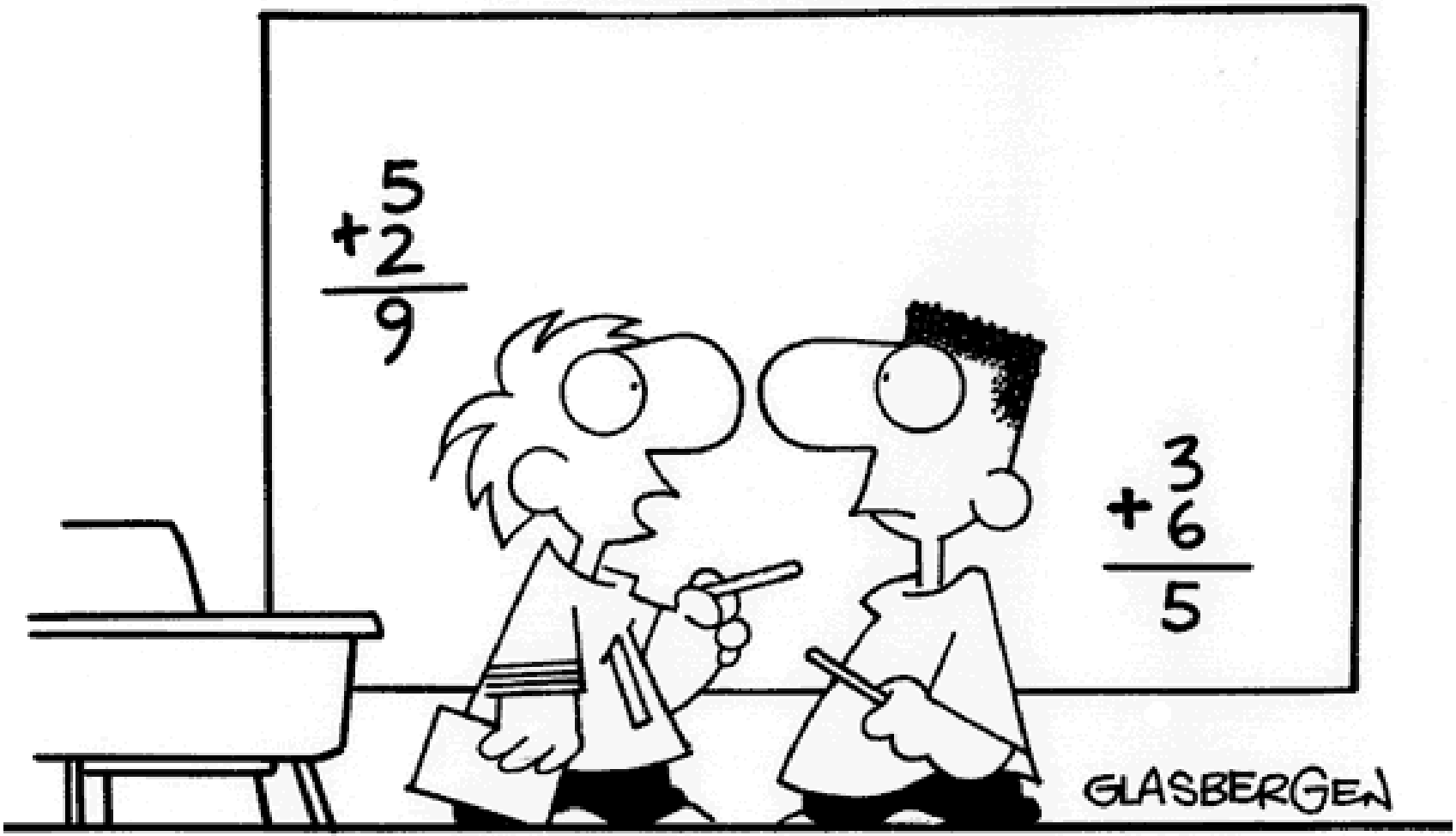

"My lawyer says I can sue the school because they're violating my right to be stupid."

Practical applications

Applying 'make it stick' to mathematics leads to the following type of implementation:

1. Teach/lecture.
2. **Tests**. (assignments, in-class tests). Compulsory.
3. **Mark**; some of marking must be done by lecturer.
4. **Hand back, pointing out errors**; this should be done for each individual.
5. if necessary more tests on same/similar material until he/she gets it right; then test again, mark, point out errors, until material is done correctly.
6. Within teaching/lecturing, practice retrieval by 'testing'.

Woe is me. Why don't people take heed?

This was tried out in a class of about 250 service (business) students. The results were astonishing. In one year failure rates were reduced from 22% to less than 11% in the *summer* (first) exam; marks in general were much higher, standards were maintained and in fact were improved.

I could not get the message across! Excuses: "Not properly sourced; no theoretical backing; too time-consuming...(leave me alone, I want to get on with my research!) ..etc., etc. ... ".

Make it stick

'make it stick' has the theory and practice which confirm that such methods work. Read it!

Further 'make it stick' implies the following:

1. Multiple choice tests are out and should not be used.
2. Lecture notes should not be made available on-line. This encourages non-participation, affects adversely class attendance and is a lazy way out for both lecturer and student.
3. Have regular homework and class tests.
4. To prevent 'cogging' of homework, have an assignment and then soon after have a class test with almost the same questions as in the assignment. People could then justifiably 'work together' but then would need to be independent for class test.

Further ...

'make it stick' confirms that 'Multiple Choice' is not a good assessment tool nor a good learning tool and should not be used. It encourages guessing, does not show how mistakes can be corrected, gives 100% or 0% and penalises student for making an error that could be corrected.

Story about Inter Cert..

If at the end of the year, 50% or so of the class fail, the problem is not with the students but with the lecturer; he/she should have been applying methods to discover errors and misunderstandings and corrected these.

Too much Emphasis

There is over emphasis in third level on 'research' or more precisely on *writing papers* and *quantity* of research as opposed to *quality of research*. Citation indices, impact factors are almost meaningless when it comes to maths at least; see article in Notices of AMS where indeed Mathematicians, 'Applied' Mathematicians and Statisticians are all agreed on this – for once!

Where does this fit into, or where could it fit into, maths' learning and support networks?

Where does this fit into mathematics' learning and support networks?

Exercise!

In conclusion

We lose something of the order of 70% of what we've just heard or seen.

After that, forgetting begins to slow and last 30% or so falls away more slowly.

Have you at this stage even retained 30% of what I've said??

Thus: Do some active retrieval! This could be done by testing yourself.

Read the book; heed it! Use 'retrieval-testing' methods on the content!